

A combined linear and nonlinear presolve for integer optimization problems

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MINLP

$$\begin{array}{ll}\min & f(x) \\ \text{s.t.} & g(x) \leq 0 \\ & x_i \in \mathbb{Z} \quad i \in I\end{array}$$

- **Sources of nonconvexity**
 - The objective/feasible space may be nonconvex
 - Integrality requirements on some variables
- **Assume**
 - f, g are recursive compositions of sums and products of univariate functions (exp, log, powers)
 - f, g are bounded

PRESOLVE LITERATURE

LP

- Brearly et al. (1975)
- Tomlin and Welch (1983)
- Vanderbei (1991)
- Fourer and Gay (1993)
- Andersen and Andersen (1995)
- Gondzio (1997)
- Ioslovich (2001)

MIP

- Johnson and Suhl (1980)
- Guignard and Spielberg (1981)
- Crowder et al. (1983)
- Hoffman and Padberg (1991)
- Savelsbergh (1994)
- Achterberg et al. (2020)

NLP/MINLP

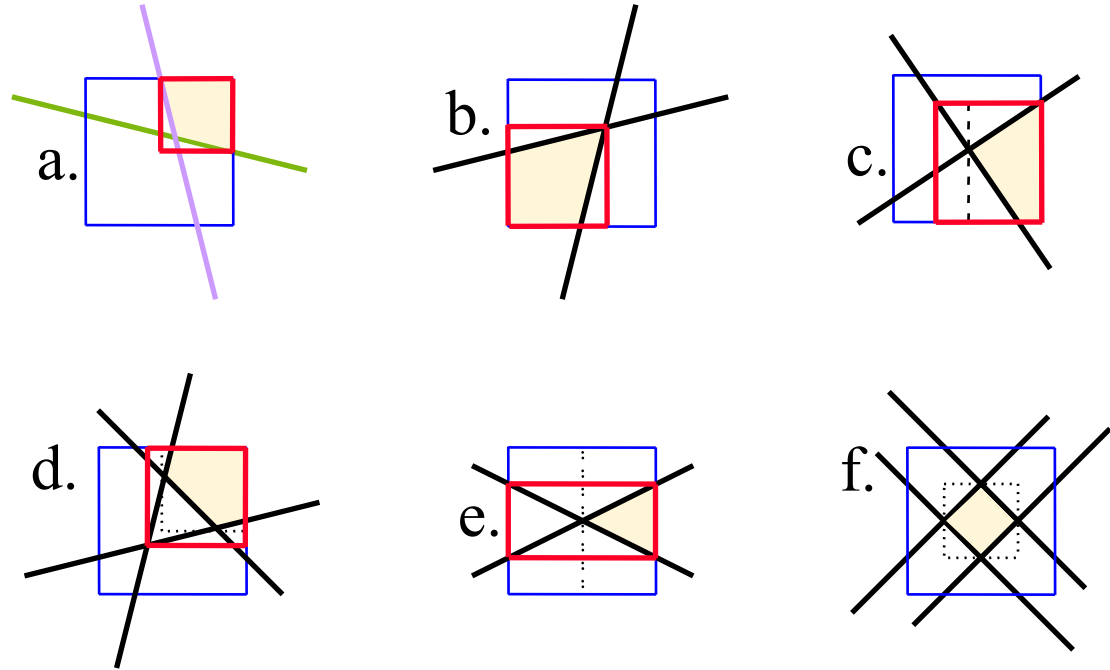
- Papalambros and Wilde (1988)
- Ryoo and Sahinidis (1995)
- Snectman and Sahinidis (1995)
- Gould and Toint (2004)

BASIC PRESOLVE

- Removal of empty rows
- Removal of empty columns
- Removal of redundant rows
- Removal of fixed columns
- Crossing of bounds on rows
- Crossing of bounds on columns
- Unboundedness

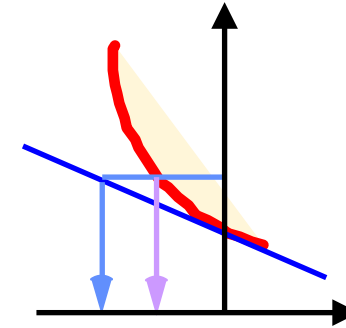
INFERENCEAL PRESOLVE

- Feasibility-based bound tightening



- Poor man's LPs
- Poor man's NLPs

- Optimality-based bound tightening



- Probing
- Removal of singleton/doubleton rows and columns
- Removal of parallel rows and columns
- Removal of dominated columns
- Monotonicity-based presolve

REFORMULATION PRESOLVE

- Substitution
- Nonzero cancellation
- Replacing integers by binary expansions
- Complete problem reformulations, such as MIBQP to MIP
- Fourier-Motzkin elimination

$$\left. \begin{array}{ll} \sum_{j \neq k} a_{ij} x_j - x_k & \leq b_i \quad i = 1, \dots, m \\ -\sum_{j \neq k} a_{lj} x_j + x_k & \leq b_l \quad l = 1, \dots, n \end{array} \right\} \Rightarrow$$

$$\left. \begin{array}{ll} \sum_{j \neq k} a_{ij} x_j - b_i & \leq x_k \quad i = 1, \dots, m \\ x_k & \leq \sum_{j \neq k} a_{lj} x_j + b_l \quad l = 1, \dots, n \end{array} \right\} \Rightarrow$$

$$\sum_{j \neq k} a_{ij} x_j - b_i \leq \sum_{j \neq k} a_{lj} x_j + b_l \quad i = 1, \dots, m; \quad l = 1, \dots, n$$

Potential explosion of the number of constraints

Kohler, 1967; Duffin, 1974; Williams, 1986; Huynh et al., 1992; Imbert, 1993; Kannippan and Thangavel, 1996

FOURIER-MOTZKIN ELIMINATION

$$\begin{array}{ll}
 \min & -x_1 + x_2 \\
 \text{s.t.} & -x_1 + x_2 - x_4 \leq -30 \quad (1) \\
 & 2x_2 + x_3 + 2x_4 \leq 50 \quad (2) \\
 & 3x_2 - x_3 + 3x_4 \leq 40 \quad (3) \\
 & x_1 \in [0, 40]; x_2, x_3, x_4 \geq 0 \quad (4)
 \end{array}
 \xrightarrow{\text{Eliminate } x_3}
 \begin{array}{ll}
 & x_3 \leq 50 - 2x_2 - 2x_4 \quad (2') \\
 & 3x_2 + 3x_4 - 40 \leq x_3 \quad (3') \\
 & 0 \leq x_3 \quad (4')
 \end{array}$$

$$\begin{array}{ll}
 \min & -x_1 + x_2 \\
 \text{s.t.} & -x_1 + x_2 - x_4 \leq -30 \quad (1) \\
 & x_2 + x_4 \leq 18 \quad (5) \\
 & x_1 \in [0, 40]; x_2, x_4 \geq 0 \quad (4'')
 \end{array}
 \xrightarrow{\text{Eliminate } x_4}
 \begin{array}{ll}
 & -x_1 + x_2 + 30 \leq x_4 \quad (1') \\
 & x_4 \leq 18 - x_2 \quad (5') \\
 & 0 \leq x_4 \quad (4''')
 \end{array}$$

$$\begin{array}{ll}
 \min & -x_1 + x_2 \\
 \text{s.t.} & -x_1 + x_2 \leq 12 \quad (6) \\
 & x_1 \in [0, 40]; x_2 \in [0, 18]
 \end{array}
 \xrightarrow{\text{Solve}} x_1 = 40, x_2 = 0 \xrightarrow{\text{Postsolve}}
 \begin{array}{l}
 x_1 = 40 \\
 x_2 = 0 \\
 x_3 \in [-40, 50] \\
 x_4 = 0
 \end{array}$$

MONOTONICITY PRINCIPLES

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g(x) \leq 0 \end{array}$$

- **Monotonicity Principle 1**

If $f(x)$ and $g(x)$ are nondecreasing w.r.t. x_i , fix x_i at its lower bound

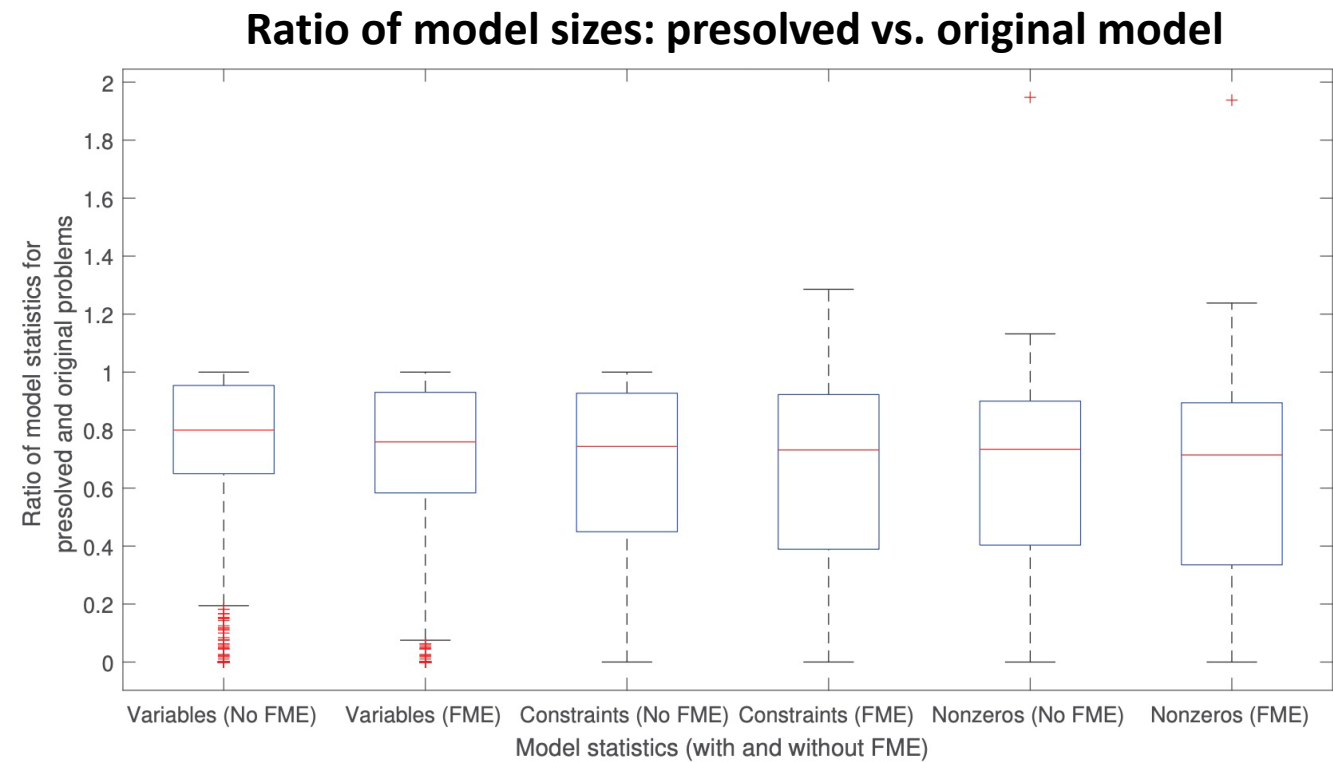
- **Monotonicity Principle 2**

For an nonobjective variable x_i , if all the constraints in $g(x) \leq 0$ are nondecreasing (nonincreasing) w.r.t. x_i , then fix x_i to its lower (upper) bound

COMPUTATIONAL ASSESSMENT OF FME

- **LP test set**
 - LP problems from Mittelmann, and Meszaros, and Netlib LP
 - LP relaxations of problems from five libraries: Globallib, Princetonlib, MINLPLib, IBMlib
 - After eliminating all infeasible and duplicate problems, 1713 problems are collected
- **All LPs solved by CPLEX**
- **Presolve strategies**
 - Nopre: no presolve
 - CPLEXpre: enable CPLEX presolve
 - BARONpreX: enable BARON presolve without FME
 - BARONpre: enable BARON presolve with FME
 - BCpreX: enable both BARON's presolve without FME, followed by CPLEX's presolve
 - BCpre: enable both BARON's presolve with FME, followed by CPLEX's presolved

FME MODEL REDUCTION



Shifted geometric means of model statistics for different presolve strategies

Model statistics	Presolve strategies					
	Nopre	BARONpreX	BARONpre	CPLEXpre	BCpreX	BCpre
Constraints	431	336	330	254	250	249
Variables	566	474	468	328	323	322
Nonzeros	1625	1195	1178	855	847	845

FME IMPACT ON SIMPLEX ITERATIONS

Shifted geometric means of Phase I iterations for different presolve strategies

Methods	The number of iterations in Phase I					
	Nopre	BARONpreX	BARONpre	CPLEXpre	BCpreX	BCpre
Default	82	43	43	32	32	29
Primal	3244	3651	3668	3178	2903	2872
Dual	1454	56	53	47	41	41

Note: In Phase I, CPLEX searches for a feasible solution; in Phase II, CPLEX searches for the optimal feasible solution.

Shifted geometric means of total simplex iterations for different presolve strategies

Methods	The total number of iterations					
	Nopre	BARONpreX	BARONpre	CPLEXpre	BCpreX	BCpre
Primal	30525	32049	32210	29483	28786	28605
Dual	19499	14547	14454	13908	13529	13449

BARON'S presolve reduces primal and dual CPLEX iterations by 3%

FME IMPACT ON CPLEX CPU TIME

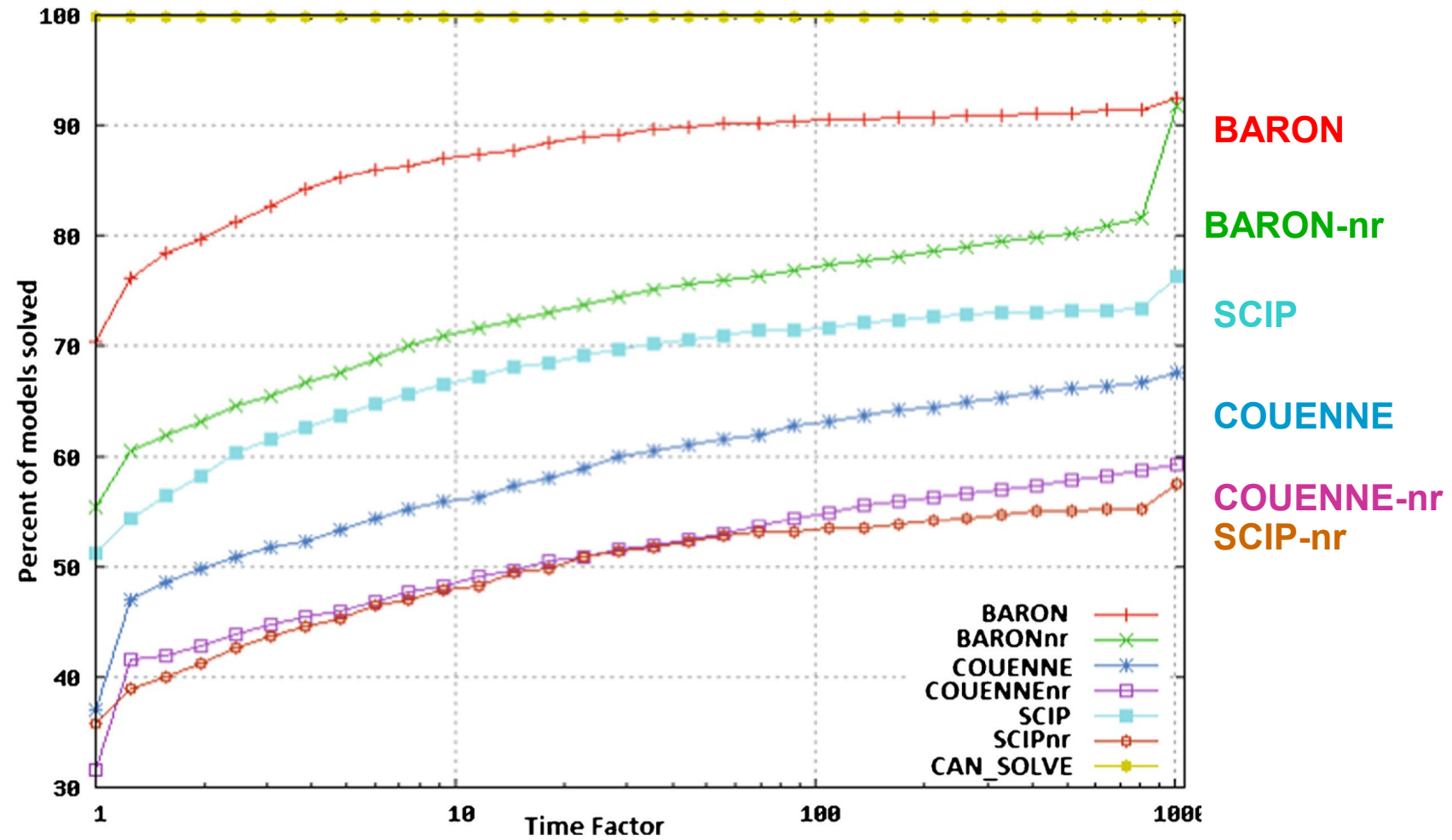
Shifted geometric means of CPLEX CPU times for different presolve strategies

Methods	Execution time					
	Nopre	BARONpreX	BARONpre	CPLEXpre	BCpreX	BCpre
Default	9.12	6.16	6.16	4.64	4.73	4.69
Primal	18.63	17.58	17.37	14.20	13.00	12.64
Dual	18.74	9.14	9.30	7.84	7.13	7.15

BARON'S presolve makes primal and dual CPLEX faster by 10%

REDUCTION BENEFITS

1740 problems from GlobalLib, MINLPLib and PrincetonLib



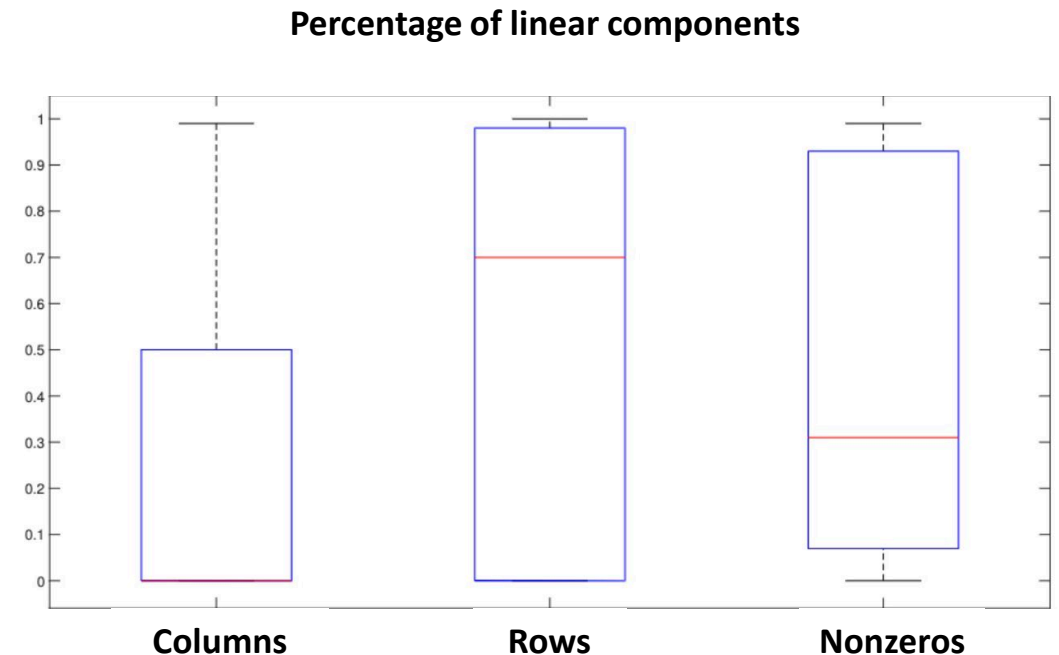
Solver performance deteriorated up to 170% when reduction is turned off

Puranik and Sahinidis (2017)

NLP TEST PROBLEM STATISTICS

- **Test set**
 - NLP problems from Globallib, Princetonlib, Netlib
 - Root-node continuous relaxations of mixed-integer nonlinear problems (MINLP) in libraries: MINLPlib, MINLPlib2, IBMlib
 - After filtering out problems that are duplicate, infeasible or unbounded and models for which bounds are not available to construct bounded factorable relaxations, 7462 problems are collected

Statistics	Rows	Cols	IntroRows	IntroCols	Nonzeros
Minimum	0	1	1	2	1
First quantile	4	20	7	133	453
Median	33	60	44	385	1700
Third quantile	127	196	157	1324	5145
Maximum	164321	107222	164322	8177833	8182013



REPRESENTATIVE EXAMPLES

Problem	Original model size			Presolved model size			Solution time (s)	
	m	n	nz	m	n	nz	Without presolve	With presolve
qplib_9030	5001	10001	57982	53	121	415	> 500	5
routingdelay_proj	2978	1124	13231	1492	1086	11703	107	5
qplib_2910	380	451	1040	75	151	514	> 500	5
concbased	355	426	990	75	151	514	> 500	7
wastewater13m1	84	383	1115	54	368	1070	> 500	3
sporttournament16	1	121	314	0	120	312	> 500	34
torsion75	4	4008	47155	0	3750	34000	> 500	3
lop97icx	87	986	1946	126	219	2046	> 500	7
powerflow0014r	197	118	757	126	107	626	> 500	90
catmix800	1600	2403	14413	2328	2401	18769	> 500	97

m, n, nz: Number constraints, variables, nonzeros

IMPACT OF PRESOLVE ON NLPs

Measures used

- 1. Size reduction
- 2. Speed-up
- 3. Decreased density of linear components
- 4. Higher quality of local solutions

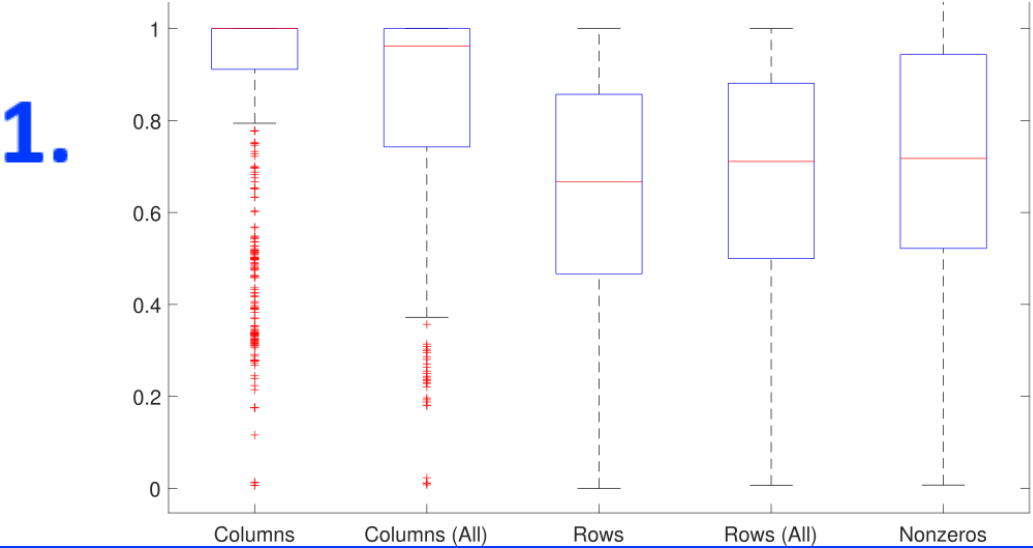
Shifted geometric means of model statistics and solution times with and without presolve for 900 presolve-impacted problems

2.

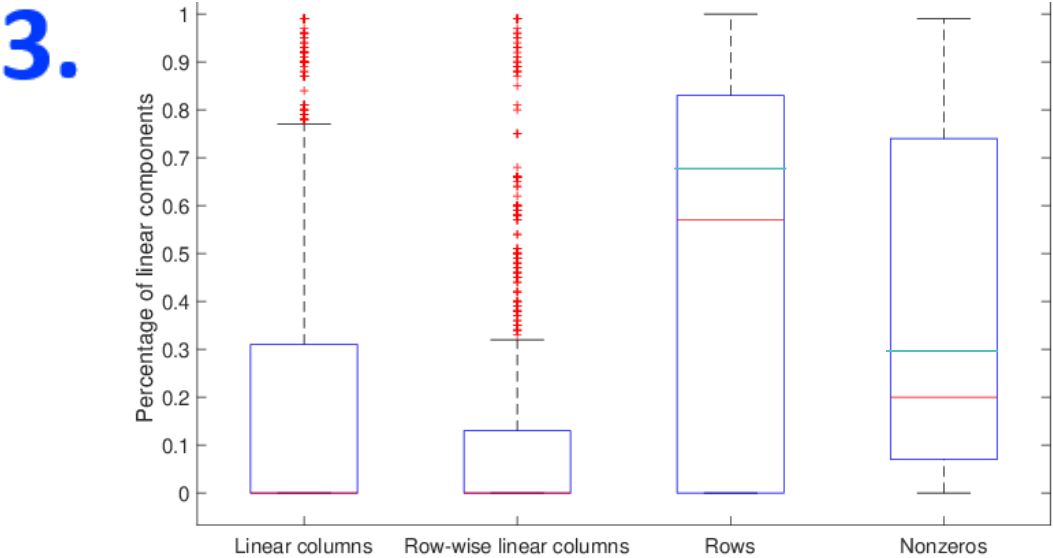
Shifted geometric means	BARONnopre	BARONpre	Reduction (%)
Number of original rows	215	160	26
Number of original and introduced rows	254	196	23
Number of original columns	200	160	20
Number of original and introduced columns	1148	930	19
Number of nonzero elements	3684	2424	34
Solution Time (s)	24	20	17

Note: The shift is 100 for the number of rows, columns, and nonzero elements; the shift is 10 seconds for the solution time.

Ratio of model statistics:
presolve vs. no presolve



Percentage of linear components

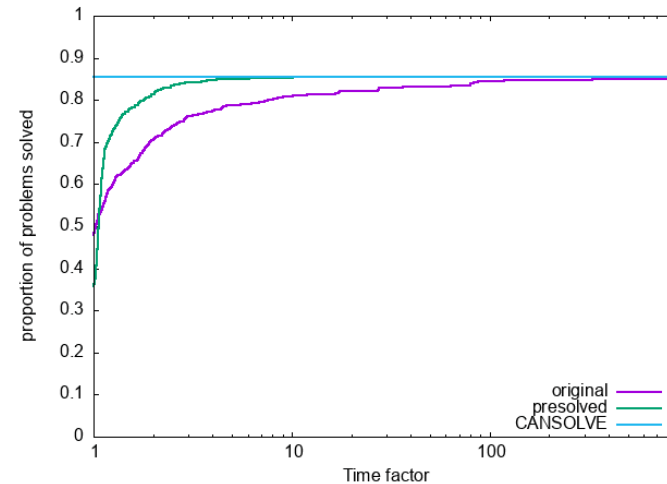


IMPACT OF PRESOLVE ON LOCAL SEARCH

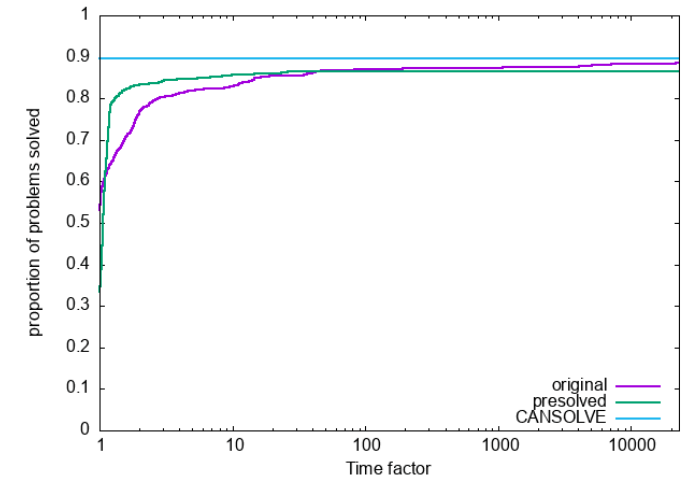
4.

Shifted geometric means of
solution times for local solvers

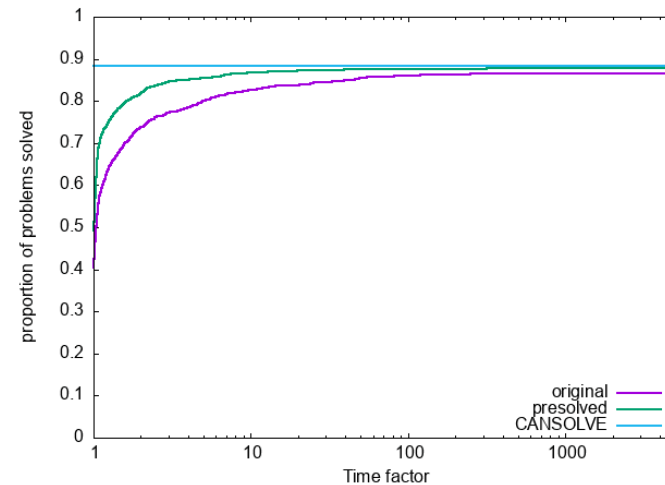
Local solver	Solution Time (s)		Improvement (%)
	Original	Presolved	
CONOPT	1.6	1.4	13
IPOPTH	4.8	4.5	6
MINOS	3.6	3.1	14
SNOPT	4.3	4.0	7



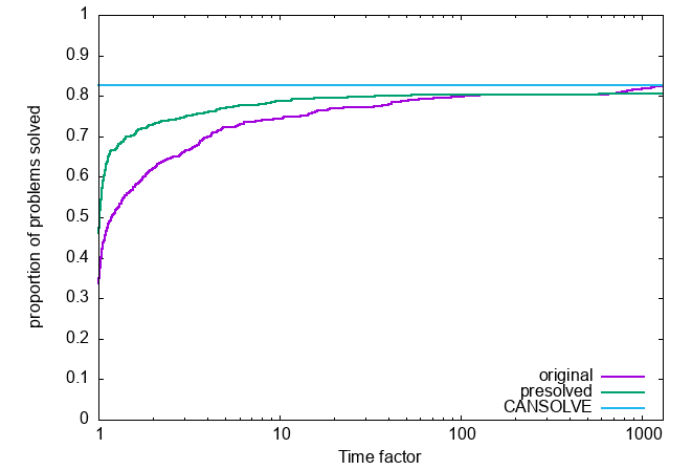
CONOPT



MINOS



IPOPTH



SNOPT

MINLP TEST PROBLEM STATISTICS

Model statistics of 3360 MINLP problems from 3802 problems in minlp.com

- Dropped 442 problems for which BARON and BARONnopre both timeout in 500 s or both finish within 1 s

Statistics	BinVars	IntVars	ConVars	Vars	Cons	IntroVars	IntroCons	Nonzeros
Minimum	0	1	1	3	0	1	0	3
First quantile	24	30	1	76	7	41	0	884
Median	50	50	63	124	83	100	0	2077
Third quantile	90	100	124	231	157	922	0	4883
Maximum	23424	23424	107209	107223	164321	2902840	2039	2920211

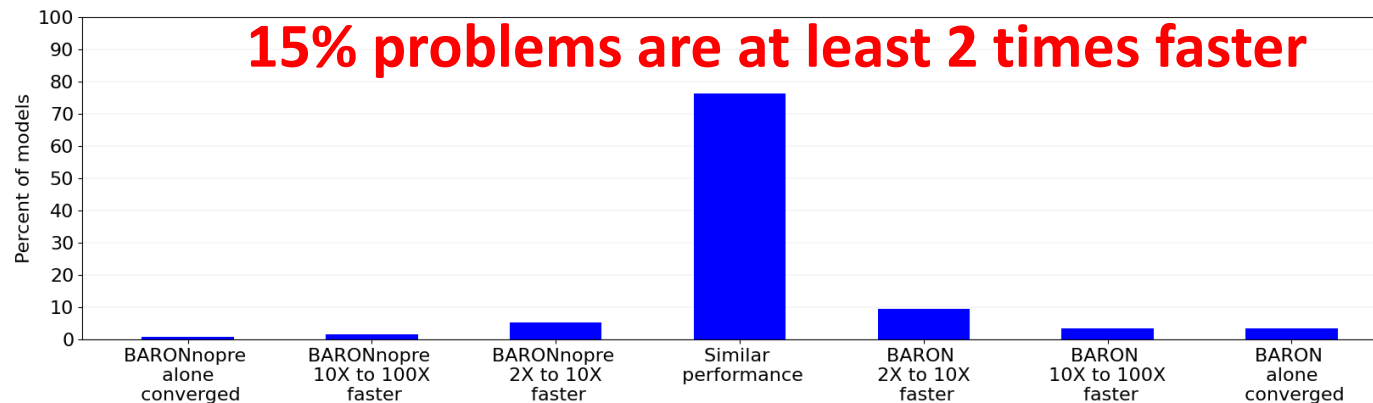
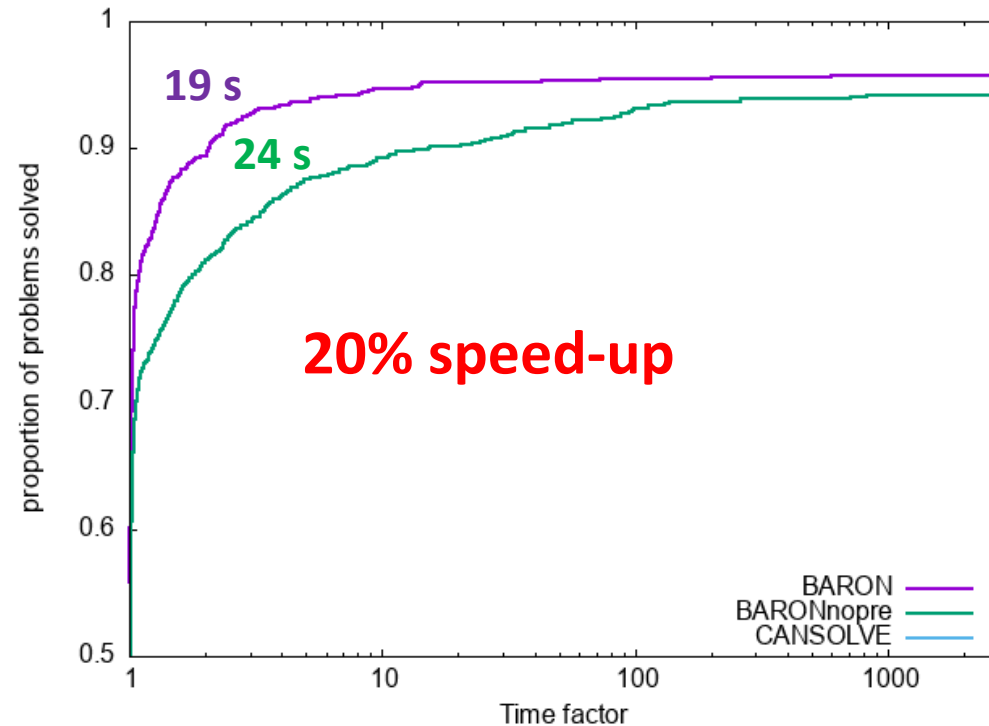
BinVars, IntVars, ConVars, Vars: Number of binary, integer, continuous, all original variables;

Cons: Number of original constraints, where the objective function is not included;

IntroVars, IntroCons: Number of introduced variables and constraints to the factorable reformulation;

Nonzeros: Number of nonzero elements in the factorable reformulation.

IMPACT OF PRESOLVE ON MINLPs



THE BARON PROJECT



- Branch-and-reduce algorithm and **BARON** software
- First publicly available software to offer **deterministic** guarantee of global optimality for mixed-integer nonlinear optimization problems
- Two-pronged approach to technology transfer
 - **Commercial**
 - Under the modeling languages GAMS, AIMMS, BARON, AMPL, MATLAB, YALMIP, Pyomo, JuMP
 - **Free**
 - Under the NEOS server for optimization
 - Over 500,000 problems solved on NEOS



Ryoo and S (1995, 1996), Tawarmalani and S (2002, 2004), Bao et al. (2009, 2015), Zorn and S (2013, 2014), Khajavirad and S (2012, 2013, 2018), Puranik and S (2017), Kılınç and S (2018), Zhou et al. (2018), Nohra et al. (2018, 2021, 2022)

HISTORIC BENCHMARK

Mittelmann MINLP test set

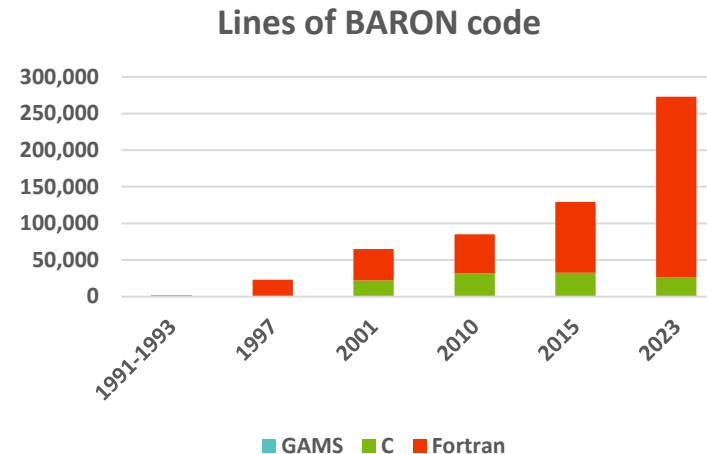
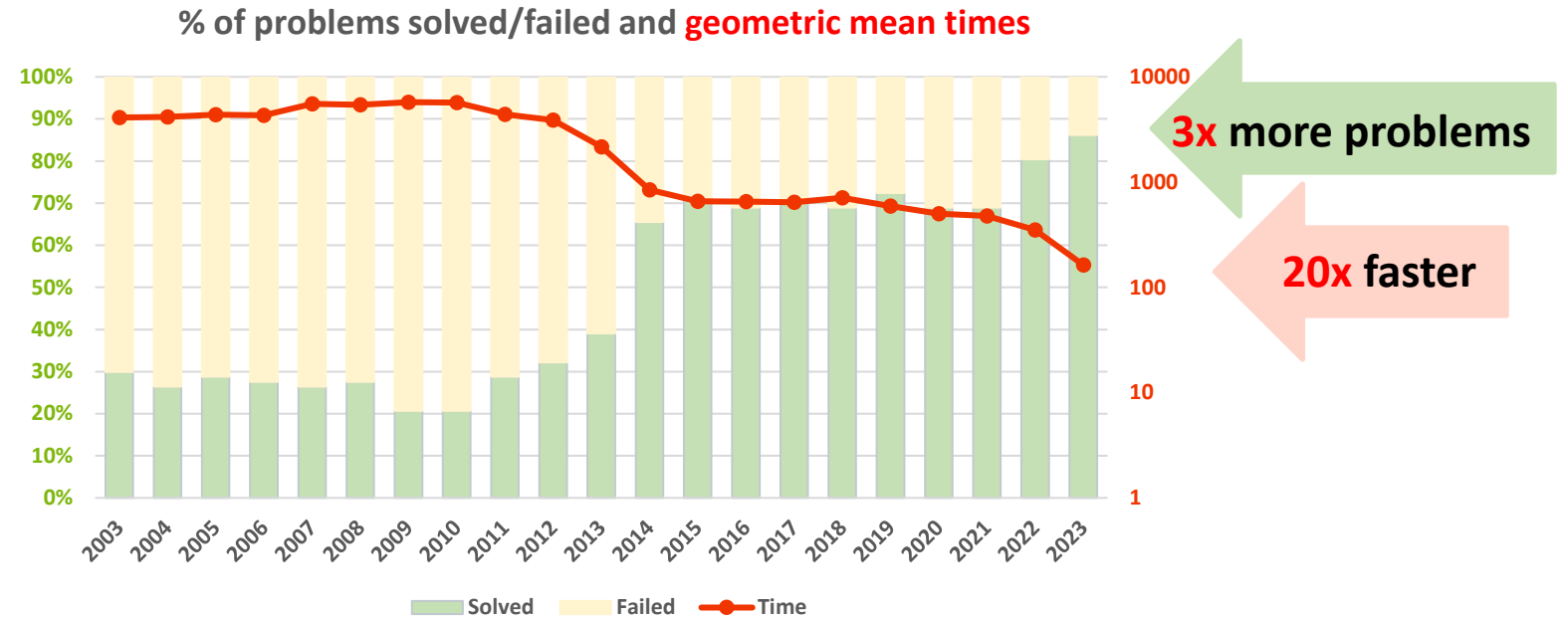
- 87 problems
- Statistics (min, max, avg)
 - Variables: 6, 107222, 2334
 - Binaries: 0, 3000, 111
 - Integers: 0, 100, 12
 - Constraints: 0, 108217, 2626

Runs under GAMS

- CPLEX, CBC
- MINOS, CONOPT, IPOPT, SNOPT, FilterSQP

64-bit Xeon X5650 2.66GHz

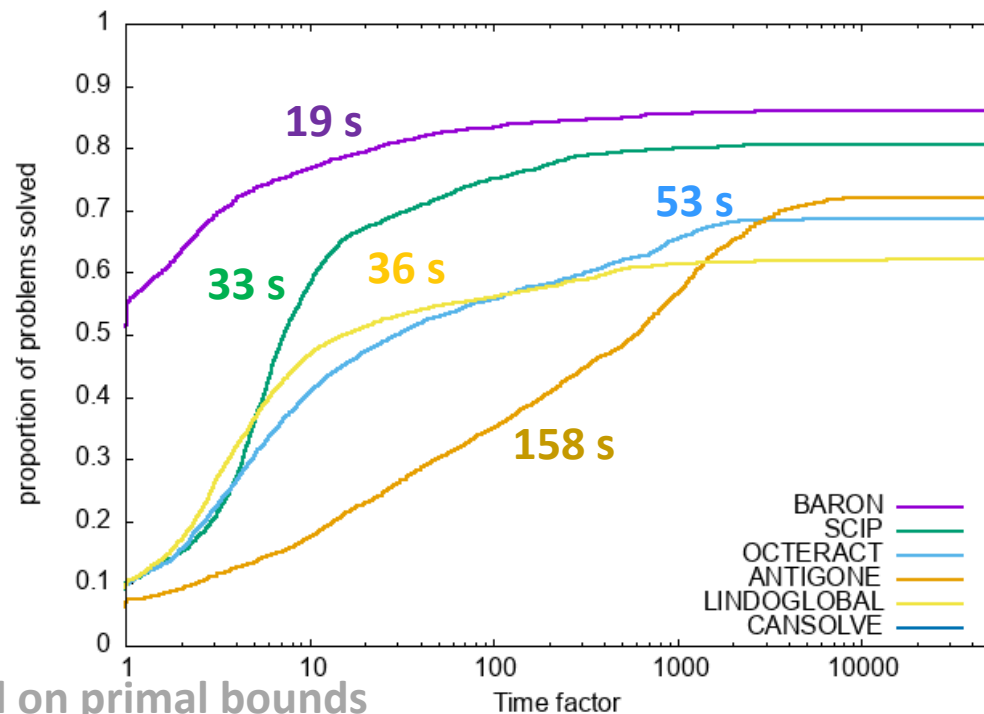
- 3600 s



STATE-OF-THE-ART GLOBAL MINLP SOLVERS

Collection

- 3802 MINLPs from minlp.com
- Drop problems for which all solvers timeout in 500 s or all solvers finish in 1 s



- 2968 problems
- Statistics (min, max, avg)
 - Variables: 3, 107223, 421
 - Binaries: 0, 8904, 115
 - Integers: 1, 10000, 123
 - Constraints: 0, 164300, 768

