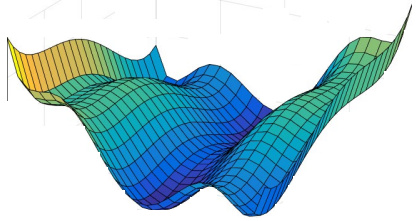

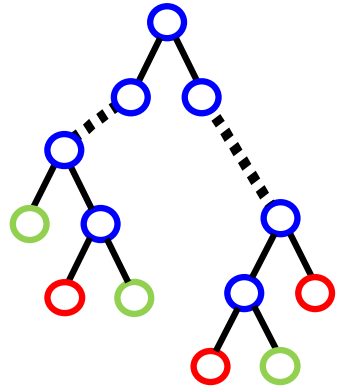

# Recent progress in the global optimization of NLPs and MINLPs with BARON

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Acknowledgements:

Anatoliy Kuznetsov, Georgia Institute of Technology

Yi Zhang, The Optimization Firm, LLC, https://minlp.com/

Georgia Tech
CREATING THE NEXT

THE OPTIMIZATION FIRM
EMPOWERING INNOVATION

# BARON HISTORY

## First generation

**1991-1993**

Duality-based range reduction

Constraint propagation

**1994-1995**

Branch-and-bound system

Finite algorithm for separable concave minimization

**1996-1997**

Parser for factorable programs

Nonlinear relaxations

Links to MINOS and OSL

## Second generation

**1998-2001**

Polyhedral relaxations

Link to CPLEX

Compressed data storage

Tree traversal, …

**2002**

Under GAMS

**2004**

Branch-and-cut

Under AIMMS

**2005-2007**

Local search

Memory management, …

## Third generation

**2008-2012**

Multi-term envelopes

Multi-variate and multi-constraint envelopes/relaxations

**2013-2016**

Links to CLP and IPOPT

Hybrid LP/NLP/MIP relaxations

Under MATLAB, YALMIP, AMPL, Pyomo and JuMP

**2017-**

NLP and MINLP Presolve

Optimality based reduction

Eigenvalue relaxations, ...

# STATUS OF THE BARON PROJECT

GAMS AIMMS AMPL THE OPTIMIZATION FIRM EMPOWERING INNOVATION MATLAB The Language of Technical Computing YALMIP JuMP PYOMO

- Branch-and-reduce algorithm and BARON software
- First publicly available software to offer deterministic guarantee of global optimality for mixed-integer nonlinear optimization problems
- Two-pronged approach to technology transfer
  - Commercial
    - Under the modeling languages GAMS, AIMMS, BARON, AMPL, MATLAB, YALMIP, Pyomo, JuMP
  - Free
    - UIUC, CMU, GT
    - Under the NEOS server for optimization

# BARON USERS


**680+ universities**

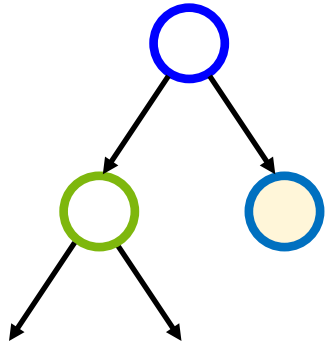
**140+ companies**

**60+ countries**

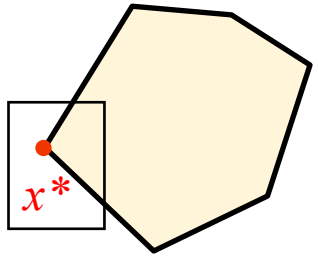
**Over 500,000 submissions through NEOS since 2001**

# ALGORITHMIC COMPONENTS

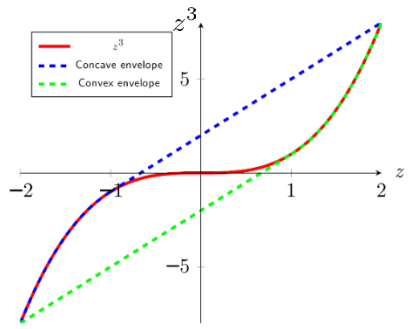
## Branch-and-bound



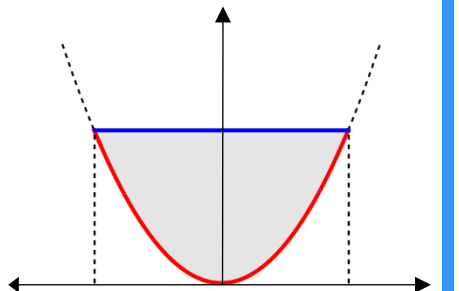
## Finiteness



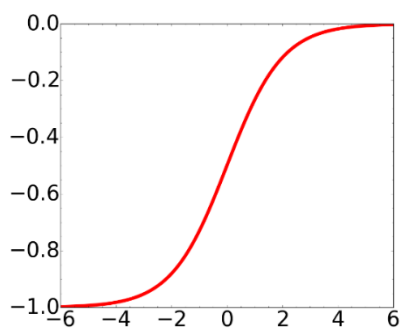

## McCormick



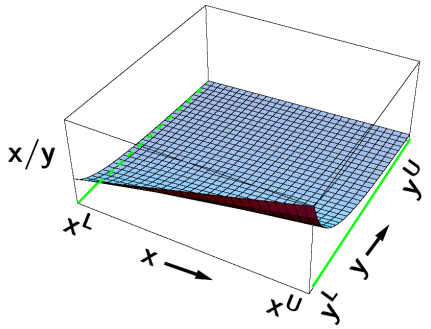

## Convexity



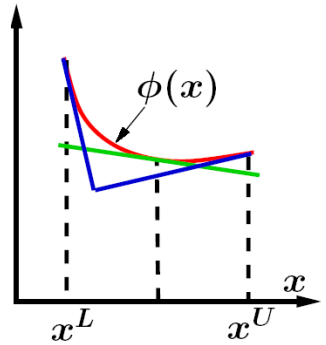
## Conv transformable



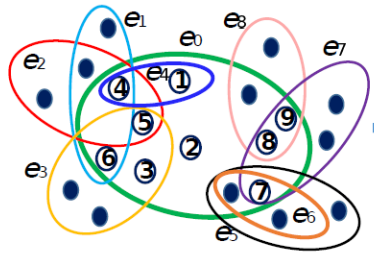

## Envelopes



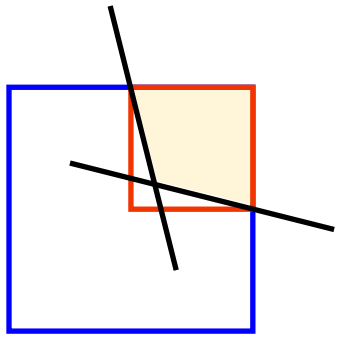

## Polyhedral



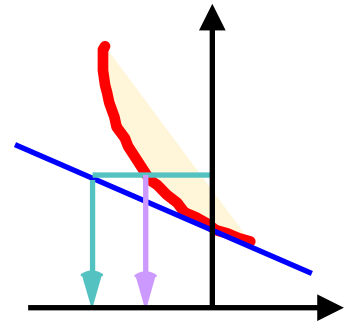

## Multilinear




## Propagation



## Duality

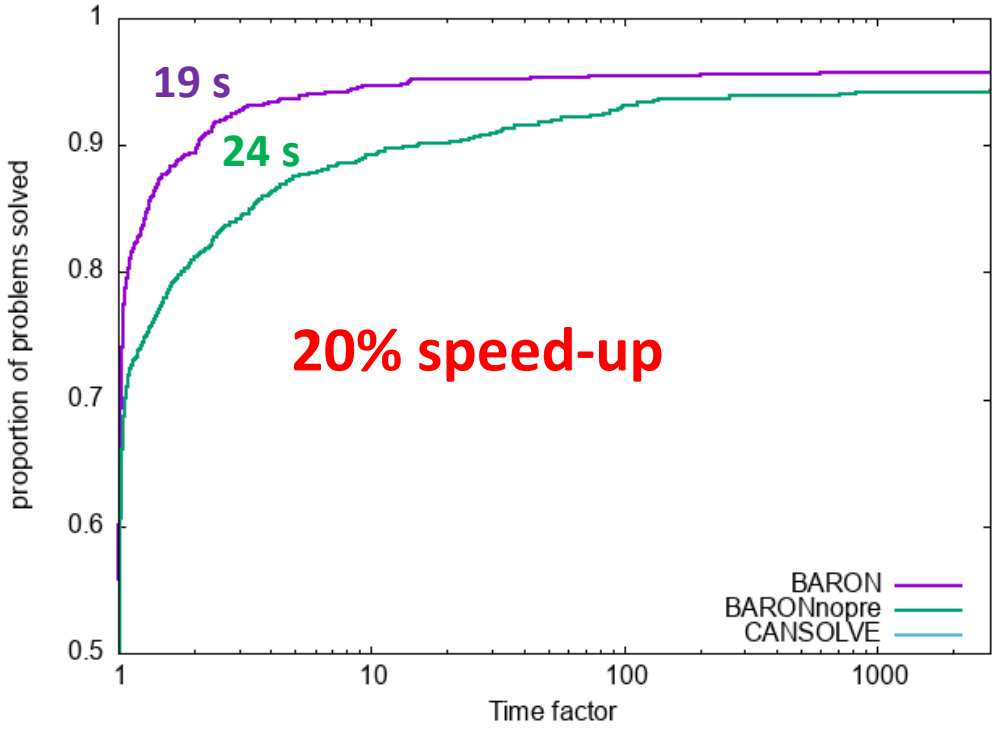


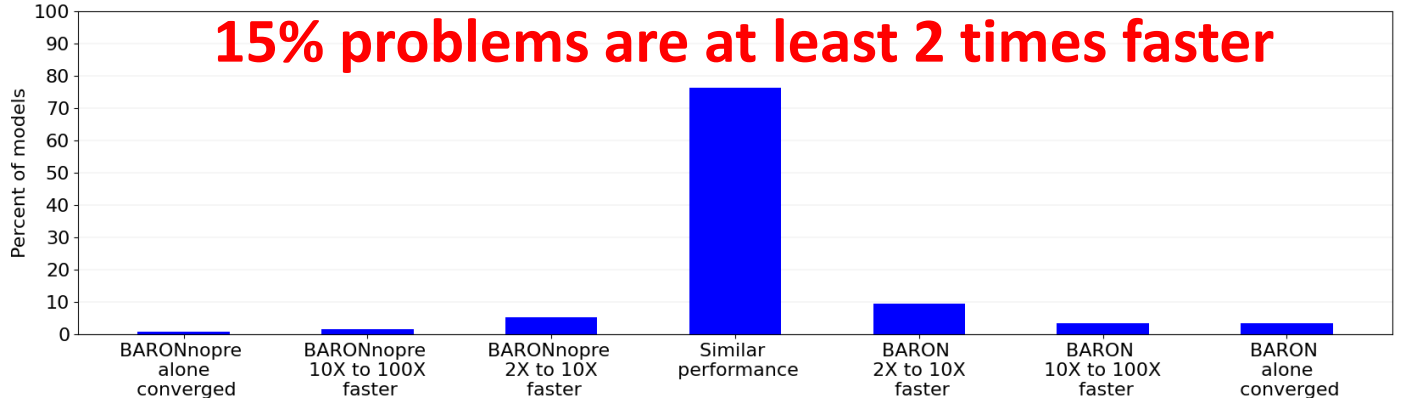
Ryoo and S (1995, 1996), Tawarmalani and S (2002, 2004), Bao et al. (2009, 2015), Zorn and S (2013, 2014), Khajavirad and S (2012, 2013, 2018), Puranik and S (2017), Kılınç and S (2018), Zhou et al. (2018), Nohra et al. (2018, 2021, 2022); Zhang et al. (2024+)

# RECENT ADVANCES IN BARON

- **Presolve**
  - Fourier-Motzkin elimination (Zhang et al., under review)
  - Combined LP/NLP/integer presolve (Zhang et al., under review)
  - Reformulations
- **Convexification**
  - Enhanced identification of convex/concave expressions
  - Simultaneous convexification
- **Hyrbid relaxations**
  - LP, NLP, MILP, QP, QCP, piecewise-linear relaxations
- **Software engineering**
  - Range reduction routines
- **Performance variability of subsolvers**
  - Safeguarding against unsafe and incorrect solutions from subsolvers
  - Learning to exploit variations among subsolvers

# IMPACT OF PRESOLVE ON MINLPs



# IMPACT OF PRESOLVE ON MINLPS

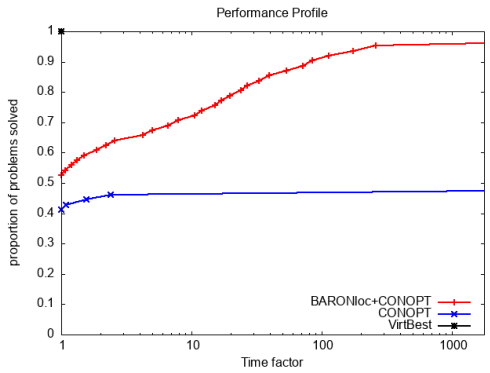
| Group of problems | | All | Solved ≥ 0 s | ≥ 1 s | ≥ 10 s | ≥ 100 s | ≥ 500 s | Time-out | |
|---|---|---|---|---|---|---|---|---|---|
| Number of Models | | 1502 | 1080 | 787 | 659 | 558 | 510 | 422 | |
| BARON default | Tlimit | 455 | 33 | 33 | 33 | 33 | 33 | NoSol | 55 |
| | Time (s) | 98 | 18 | 30 | 40 | 48 | 49 | $Gap^{0.1}$ | 72 |
| | Nodes | 107 | 38 | 70 | 80 | 80 | 82 | $Gap^{0.01}$ | 41 |
| No presolve | Tlimit | 852 | 430 | 430 | 430 | 430 | 430 | NoSol | 66 |
| | MWorse | 683 | 683 | 588 | 540 | 484 | 452 | MWorse | 169 |
| | Worse | 101 | 101 | 48 | 30 | 17 | 14 | Worse | 58 |
| | Better | 59 | 59 | 25 | 12 | 6 | 4 | Better | 13 |
| | MBetter | 164 | 164 | 102 | 65 | 41 | 35 | MBetter | 52 |
| | Time (s) | 400 | 165 | 496 | 1027 | 1946 | 2415 | $Gap^{0.1}$ | 33 |
| | Nodes | 3486 | 2502 | 15372 | 45246 | 121518 | 195484 | $Gap^{0.01}$ | 16 |
| Impact (%) | Tlimit | 47 | 92 | 92 | 92 | 92 | 92 | NoSol | 17 |
| | Time (s) | 76 | 89 | 94 | 96 | 98 | 98 | $Gap^{0.1}$ | 118 |
| | Nodes | 97 | 98 | 100 | 100 | 100 | 100 | $Gap^{0.01}$ | 156 |

Note:

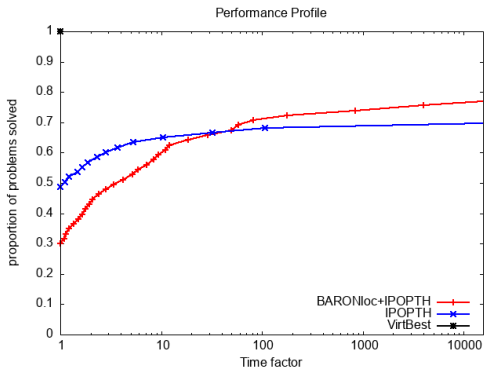
- '>= n s': a bracket collecting examples that at least one solver can solver but the slower solver costs at least n seconds
- Tlimit: number of time-out cases in each group
- Time and Nodes: shifted geometric means (shift = 10) for the solution time and number of nodes for each group
- $Gap^{0.01}$: number of time-out problems that converge to an (absolute or relative) optimality gap within 1%
- $Gap^{0.1}$: number of time-out problems that converge to an (absolute or relative) optimality gap between 1 and 10%
- MWorse = much worse; MBetter = much better; NoSol = not solved

# SPEEDING UP LOCAL NLP CODES

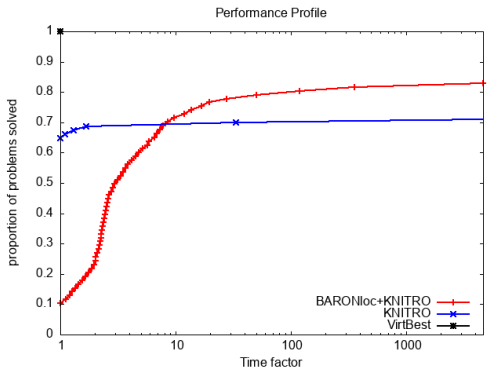
- **BARONloc: a local version of BARON that terminates once the first local solution is found**
- **BARONloc+X: BARONloc equipped with only local subsolver X**



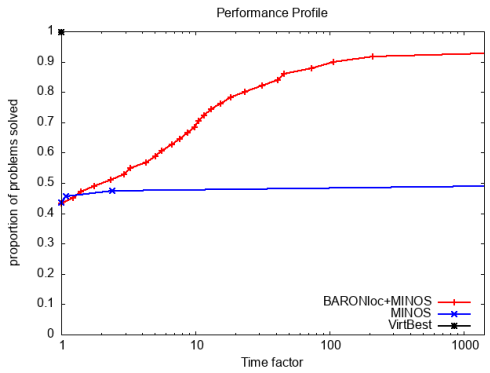

CONOPT



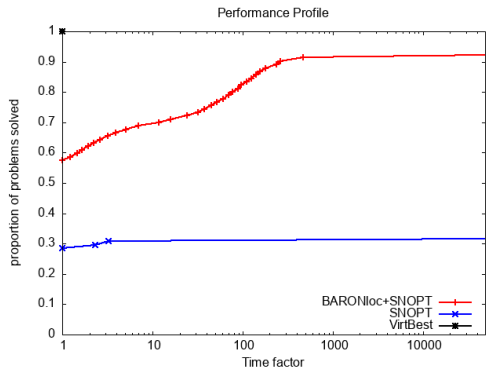

IPOPTH




KNITRO




MINOS




SNOPT

| Local solvers | CONOPT | IPOPTH | KNITRO | MINOS | SNOPT |
|---|---|---|---|---|---|
| Solved problems | 609 | 612 | 785 | 515 | 887 |
| Time (LocSolver) | 167 | 24 | 6 | 133 | 439 |
| Time (BARONloc+LocSolver) | 6 | 18 | 11 | 14 | 5 |
| Ratio | 27.8 | 1.3 | 0.5 | 9.5 | 87.8 |

# MITTELMANN MINLP BENCHMARK

## Mittelmann MINLP test set

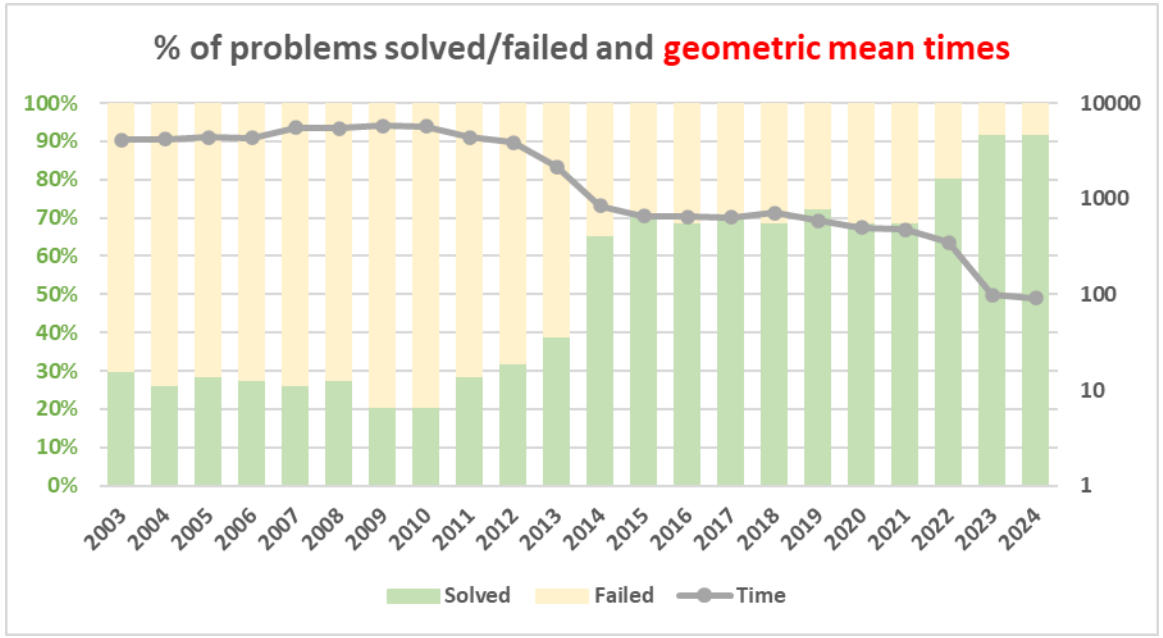
- 87 problems
- Statistics (min, max, avg)
  - Variables: 6, 107222, 2334
  - Binaries: 0, 3000, 111
  - Integers: 0, 100, 12
  - Constraints: 0, 108217, 2626

## Runs under GAMS

- CPLEX, CBC
- MINOS, CONOPT, IPOPT, SNOPT, FilterSQP

## 64-bit Xeon X5650 2.66GHz

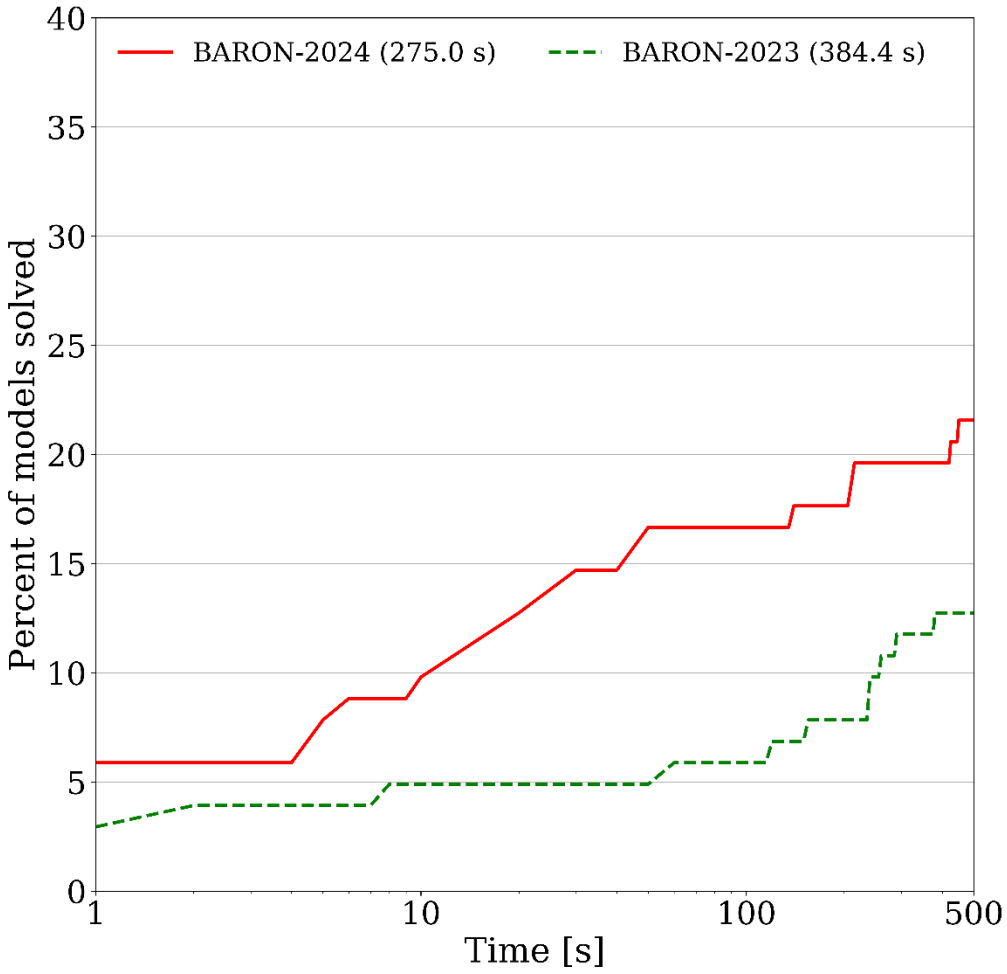
- 3 hours
- Single thread



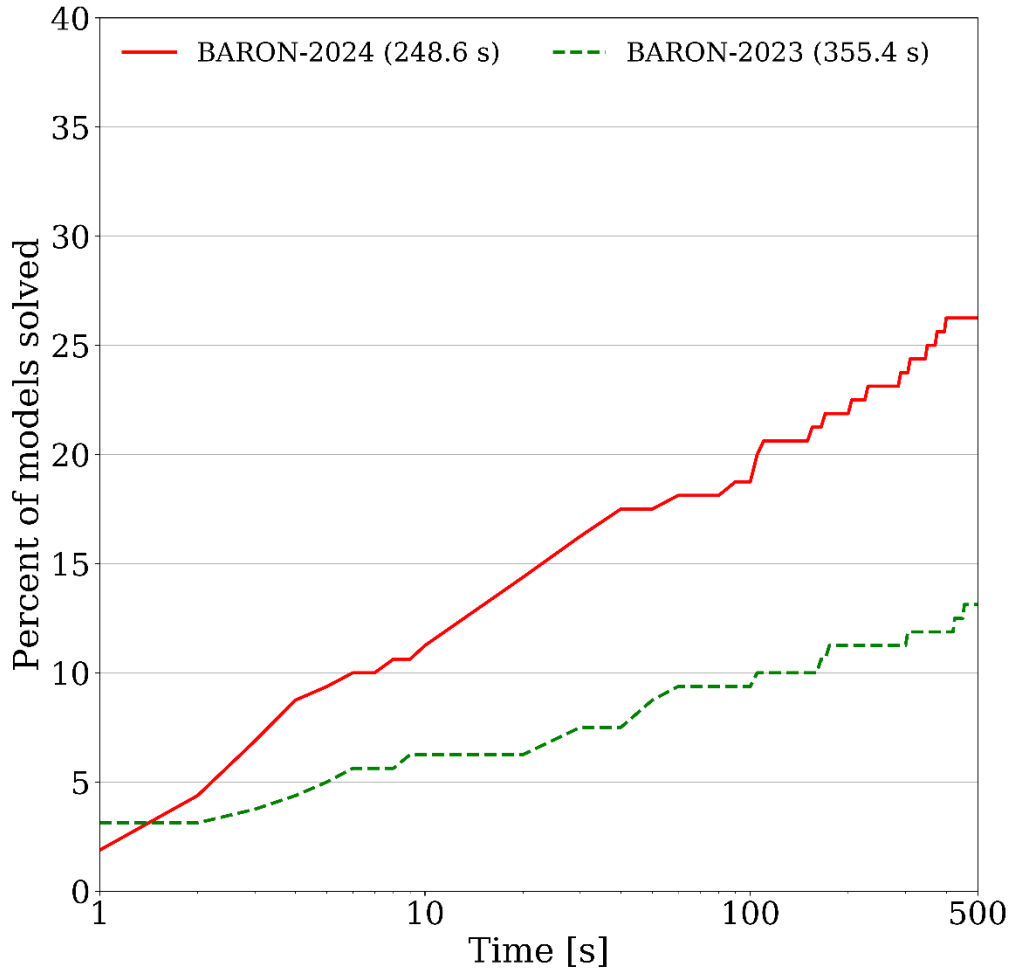

3x more problems

40x faster

# MITTELMANN NONCONVEX QCQP BENCHMARK

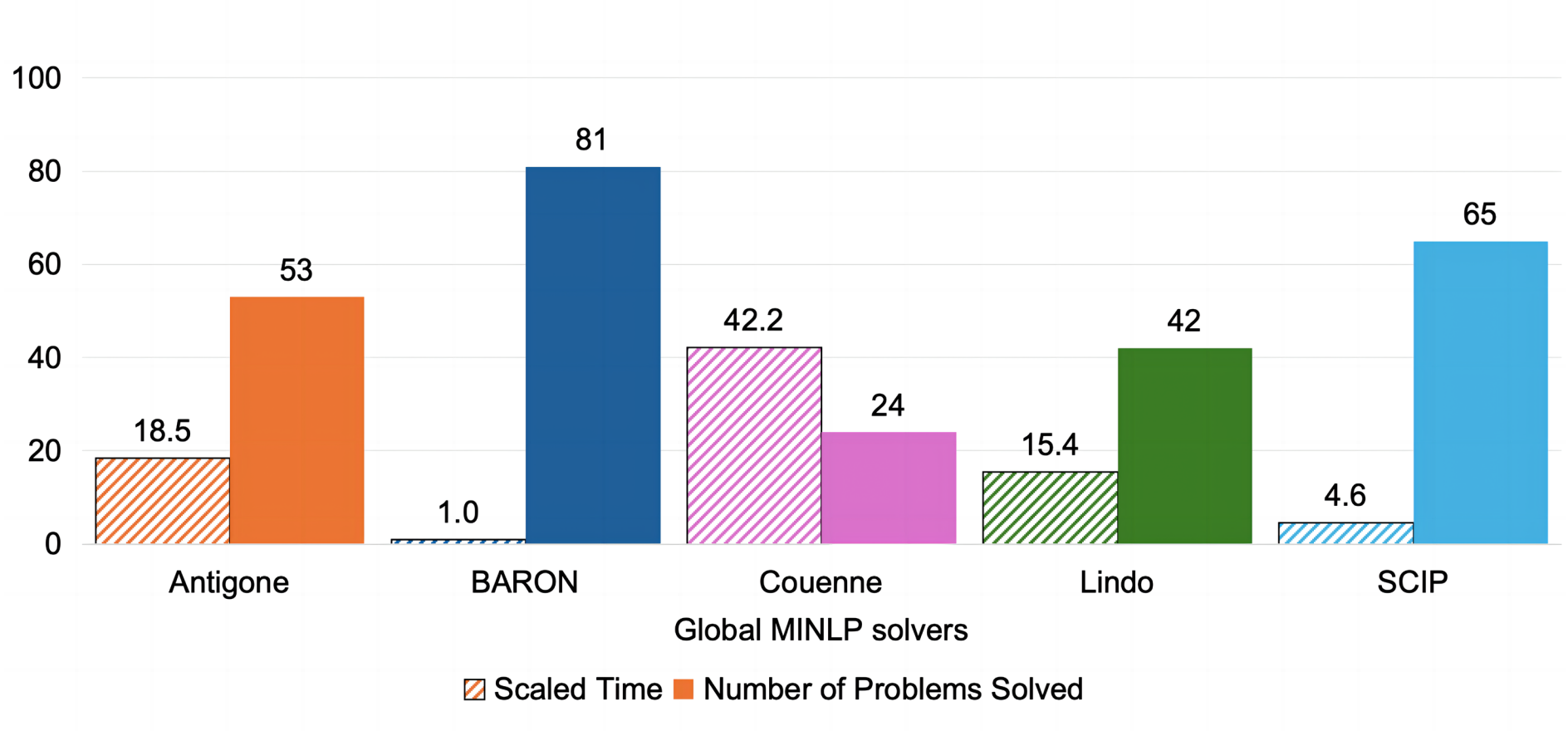



**Discrete**




**Continuous**

# GLOBAL MINLP SOLVERS ON MITTELMANN BENCHMARK

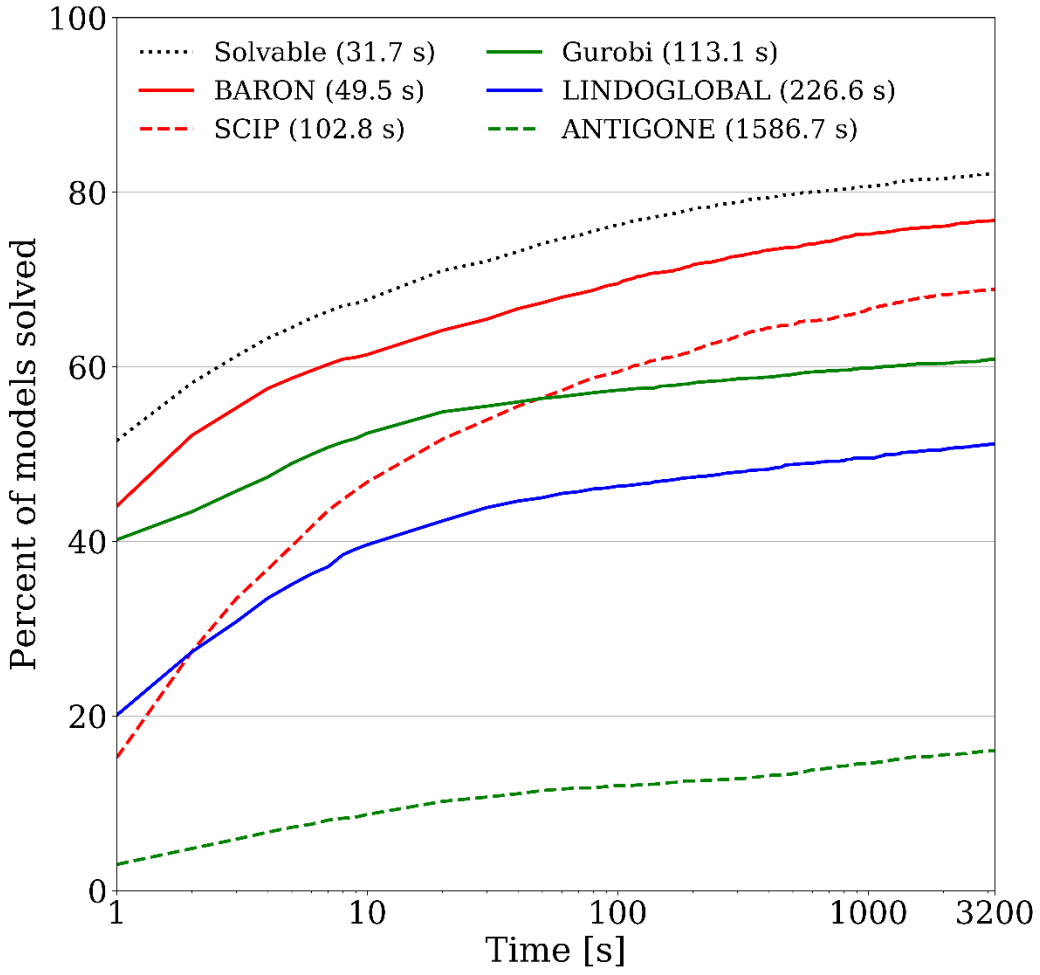



Source of data: Mittelmann, H. (2024, June 8). Mixed Integer Nonlinear Programming Benchmark (MINLPLIB) [Dataset]. Arizona State University. https://plato.asu.edu/ftp/minlp.html.

# STATE-OF-THE-ART GLOBAL MINLP SOLVERS

- **3,875 instances**
- **Average size:**
  - 784 variables
  - 208 integer variables
  - 1,241 constraints
- **Single core under GAMS/Examiner**
- **82% solvable by any solver within one hour**
- **Solvable instance size averages:**
  - 424 variables
  - 98 integer variables
  - 594 constraints
- **https://minlp.com/optimization-test-problems/**

3,875 MINLPs from minlp.com

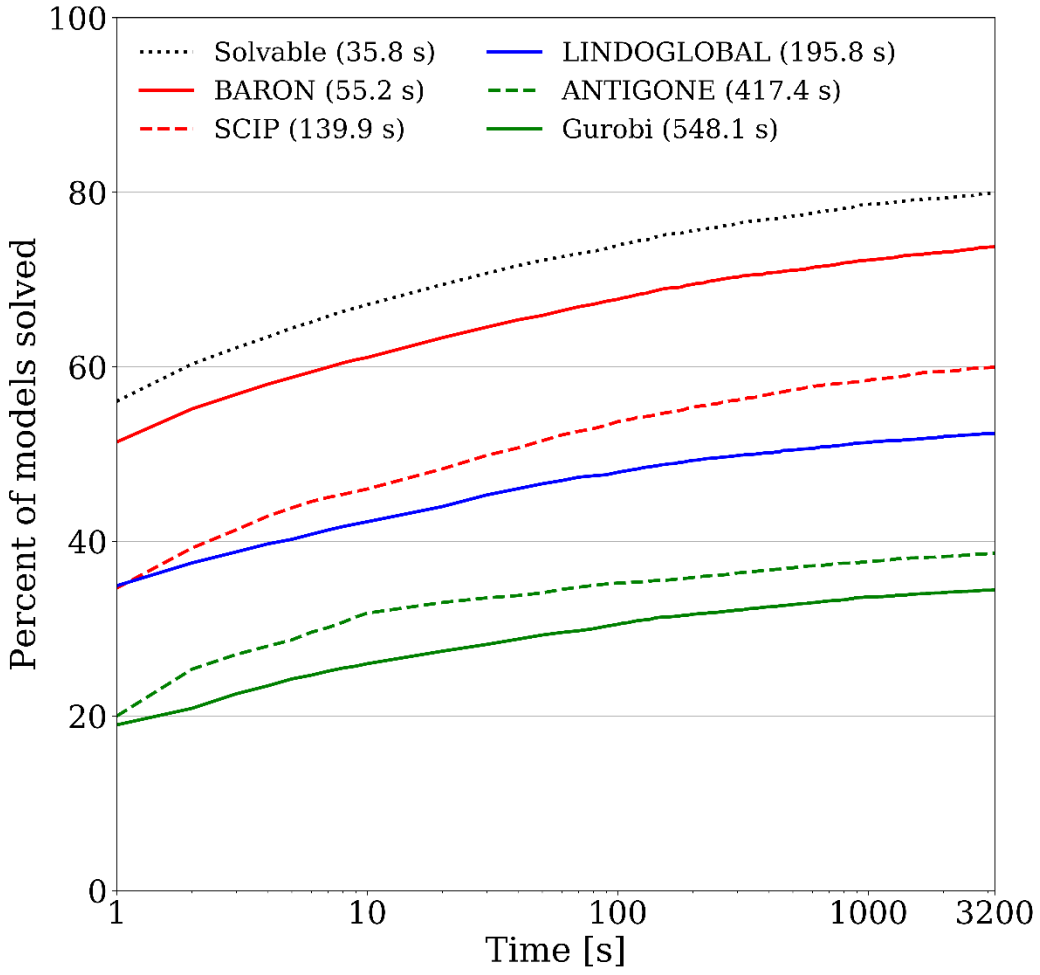



Results with Xpress were not included at the request of FICO

# STATE-OF-THE-ART GLOBAL NLP SOLVERS

- 7,787 instances
- Average size:
  - 773 variables
  - 914 constraints
- Single core under GAMS/Examiner
- 80% solvable by any solver within one hour
- Solvable instance size averages:
  - 587 variables
  - 627 constraints
- `https://minlp.com/optimization-test-problems/`

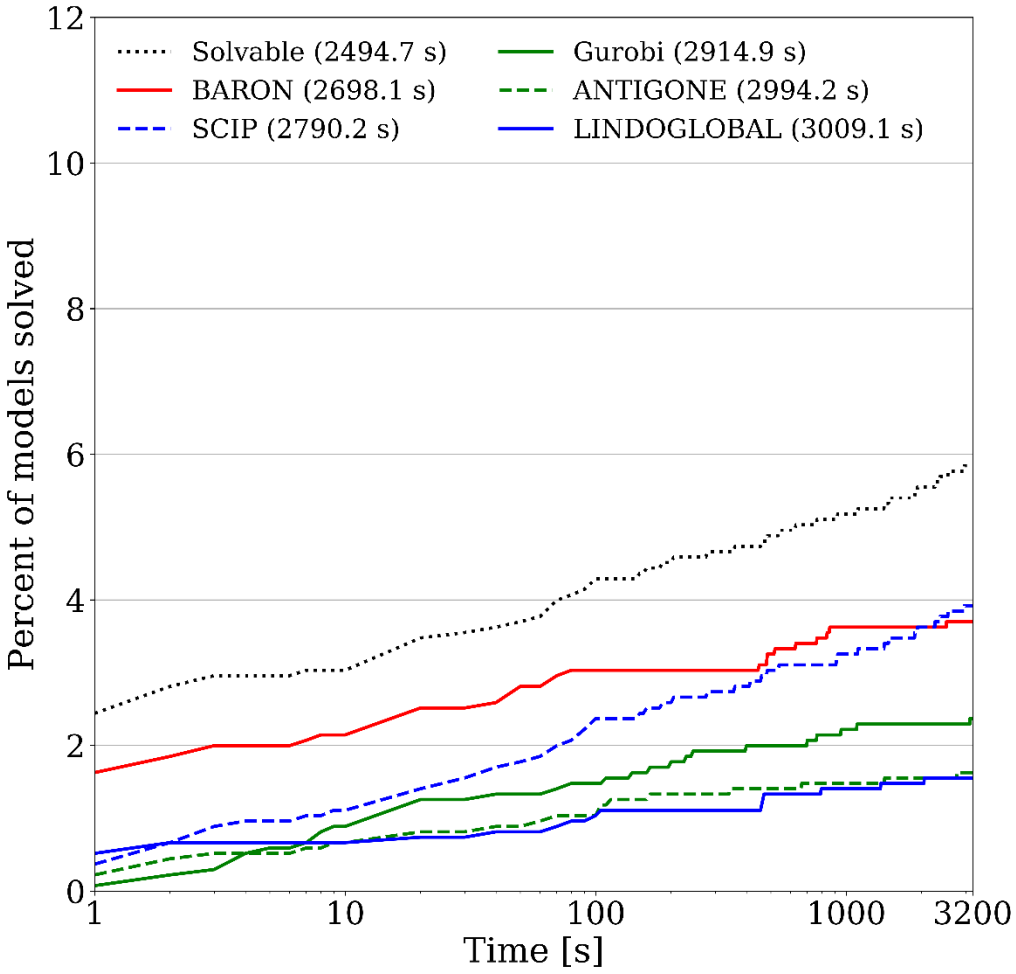
7,787 NLPs from minlp.com




Results with Xpress were not included at the request of FICO

# EuclidLib

- 1,352 instances
- Average size:
  - 6,205 variables
  - 11,718 constraints
- Single core under GAMS/Examiner
- 5.8% solvable by any solver within one hour
- Solvable instance average size:
  - 39 variables
  - 448 constraints
- `https://github.com/anatoliy-kuznetsov/EuclidLib`

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